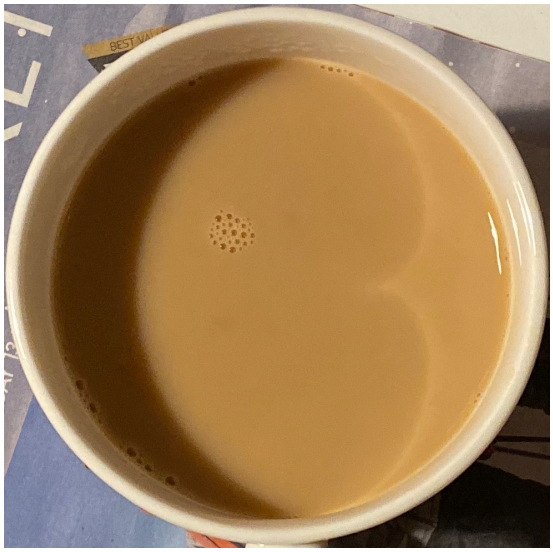
Cardioids in Tea Cups

If you are lucky, sometimes the light can interact with the surface of a cup of tea and produce a rather pleasing shape:



This is commonly referred to as a cardioid - but is it?!

With Desmos, we can easily fit a mathematical curve to a picture, but let's explore the different representations of a cardioid first.

The "easiest" is probably to work in polar form. In a 2D polar coordinate system, a cardioid can be expressed as $r = a(1 - cos(\theta)), 0 \leq \theta \leq 2\pi.$

Question: Using the mappings $x = r\cos(\theta)$ and $y = r\sin(\theta)$ show that the parametric form of a cardioid is:

$$x(t) = a\cos(t)(1 - \cos(t))$$
$$y(t) = a\sin(t)(1 - \cos(t))$$

Hint: θ and t can be interchanged as they are just dummy variables.

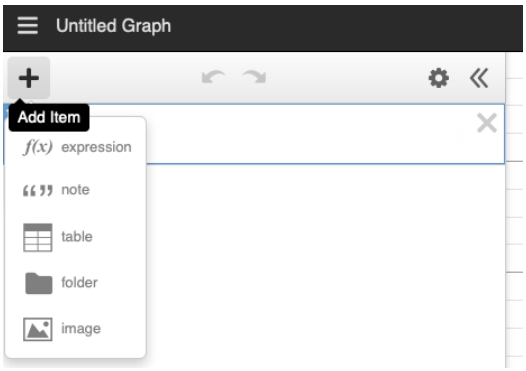
Question: Show that $(x^2 + y^2 + ax)^2 = a^2(x^2 + y^2)$ is the Cartesian form of a cardioid.

Using Desmos to Investigate the Pattern in a Tea Cup

First please download the picture above from my website at the following address: http://bit.ly/CardioidPic and save it somewhere.

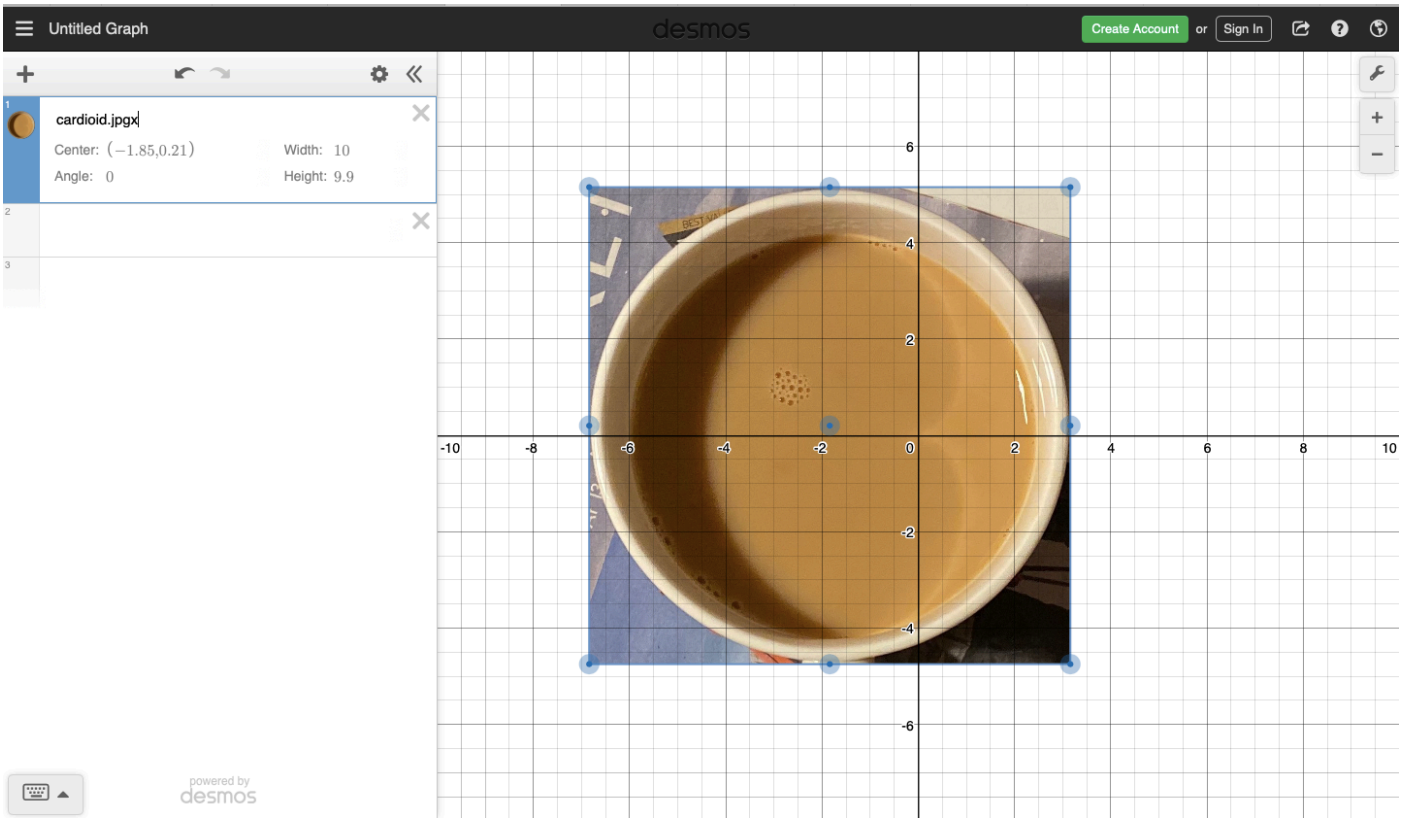
Now go to www.desmos.com and choose to “Start Graphing”.

In Desmos you can use the “add item” functionality to insert the picture on to the page.



Once you have inserted the picture, move it so that the cusp of the “tea cardioid” lies at the origin.

You will now have something that looks like this:



Then choose one of the forms discussed on page 1 and plot the curve, making sure to select to add a slider for the parameter.

Vary the parameter to obtain a good as fit to the curve as possible.

Would you say the pattern in the cup is a cardioid?