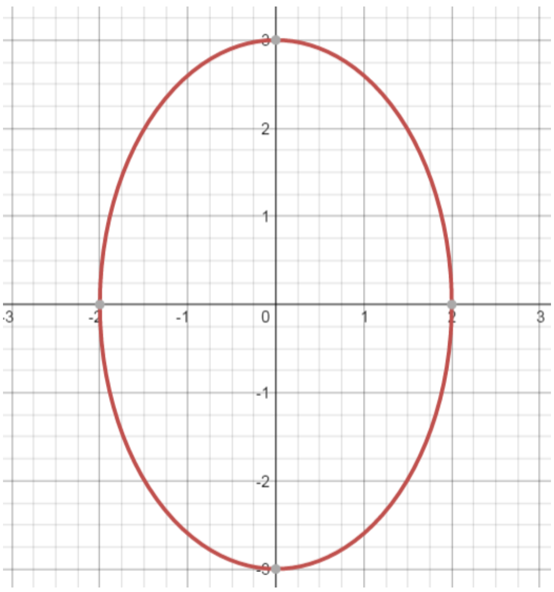
The Ellipse

1) State the parametric equations of the ellipse and derive the Cartesian equation for the ellipse.

2) By putting into standard form sketch the ellipse E given by the equation

$$4x^2 + 9y^2 = 36$$

3) Identify the values for a and b for the ellipse below and hence state the equation.



4) Find $\frac{dy}{dx}$ from both the parametric form of the ellipse and the Cartesian form of the ellipse.
Show how the parametric form could be derived from the Cartesian form if so wished.

5) Using your answers to question 4, or otherwise, find the general form for the equation of a tangent to the ellipse at the point $P(a\cos(t),\ b\sin(t))$.

6) Similarly to question 5, find the general form for the equation of the normal to the ellipse at the point $P(\ a\cos(t), b\sin(t))$.

7) Show that for the line $y = mx + c$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ the condition below must be satisfied:

$$a^2m^2 + b^2 = c^2$$

Hint: consider the discriminant of a quadratic.

8) The point $P\left(2, 3\frac{\sqrt{3}}{2}\right)$ lies on the ellipse E with parametric equations $x = 4\cos(\theta), y = 3\sin(\theta)$.

a) Find the value of θ at the point P.

b) The normal to the ellipse at P cuts the $x - axis$ at the point A. Find the coordinates of the point A.