Method of Differences Homework

Literacy	Research	Memory
method of differences, partial fractions.	Research the use of the method of differences for summing convergent infinite series.	If $u_r \equiv f(r+1) - f(r)$ then $\sum_{1}^{n} u_r = f(n+1) - f(1)$

Skills

1. By considering $f(r) - f(r+1)$ where $f(r) = \frac{r+2}{r(r+1)}$ or otherwise, find the sum of the following series
$$\sum_{r=1}^{n} \frac{r+4}{r(r+1)(r+2)}$$

2. Using the method of differences show that
$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+2)} = \frac{n}{n+2}$$

3. Use the identity $(r+1)^3 - r^3 \equiv 3r^2 + 3r + 1$ to find $\sum_{r=1}^{n} r(r+1)$

4. Find $\sum_{r=1}^{n}(2r+1)$ using an appropriate function and the method of differences.

Stretch

1) By considering the function $f(r) = r!$ Find the sum of the first $2n$ terms of the series
$$1 \times 1! + 2 \times 2! + 3 \times 3! + 4 \times 4! + \cdots$$

2) Let $f(r) = \cos(2r\theta)$. Simplify $f(r) - f(r+1)$. Use your result to find the sum of the first n terms of the series
$$\sin(3\theta) + \sin(5\theta) + \sin(7\theta) + \cdots$$