AQA A-Level Further Mathematics (Mechanics & Statistics) Warmup - Paper 1 2019

How do you calculate work done for a constant force F? How about for a variable force F	A CRV X, has probability density function given by $f(x) = \begin{cases} 3x^a; & 0 \leq x \leq 1 \\ 0; & \text{otherwise} \end{cases}$ Find the constant a and the median value M, of X.	The expected value of a function $g(X)$ of a discrete random variable X is given by:	What is the test statistic for a Chi Squared test? What modification is needed for a 2×2 contingency table?	A body moving on a horizontal circular path of radius r with a constant angular velocity has: speed - acceleration - centripetal force -
Random events occur at a rate of 3 per minute. a) Write the probability density function $f(t)$ and the cumulative density function $F(t)$ for the random variable T, the waiting time in minute between events. b) What is the mean and variance of T.	What is a p % confidence interval?	A particle of mass 1.2 kg is acted on by a time dependent force of $F = 2t + 3e^{-2t}$. Find the impulse exerted by this force if the force is applied for 2 seconds.	When would you use a $t-$test? And what is the formula for the test statistic?	How do you decide if a shape on an inclined plane will topple or slide?
What are Type I and Type II errors?	One end of a light inextensible string is attached to a point A and the other end to a particle of mass 2 kg. The string is inclined at $30°$ to the vertical.The particle moves in a horizontal circle with angular speed $\omega = 2\pi$ rad s^{-1}. Find the radius of the circle.	A particle P of mass 1kg is moving at a speed of 5ms^{-1} collides with a particle Q of mass 2kg which is at rest. Given that after the collision P moves with speed 2ms^{-1}, find the speed of Q after the collision.	A car of mass 1200 kg is moving down a hill inclined at an angle θ where $\sin(\theta) = \dfrac{1}{30}$. The car is accelerating at 1.2ms^{-1} and the engine is working at a constant rate of 35 kW. Find the magnitude of the non-gravitational resistance to motion at the instant when the car is moving travelling at 5 ms^{-1}.	Prove that the exponential distribution $f(x) = \lambda e^{-\lambda x}$, with $x \geq 0$ has a mean of $\dfrac{1}{\lambda}$.
A Geiger counter detects radioactive decays at a mean rate of 20 per minute. Find the probability that in a given, randomly chosen minute, there are i) 23 decays ii) More than 25 decays	If X and Y are normal random variables then what is the distribution of $X + Y$?	For the rectangular distribution $X \sim \text{Rect}(a, b)$ what is the probability density function, the mean and the variance?	An elastic string has natural length of 6m. If it is stretched by an extension x_1 it reaches point A, if it is stretched by an extension x_2 it reaches point B. If the modulus of elasticity is 50N find the work done in stretching the string from A to B.	In the topic of collisions how do you define the coefficient of restitution?

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Constant force: Work done - Force times perpendicular distance ($W = F \times d$) Variable Force: $W = \int_0^d F \ \mathrm{d}x$	$\int_0^1 3x^a \ \mathrm{d}x = 1 \quad \Rightarrow \quad a = 2$ $\int_0^M 3x^2 \ \mathrm{d}x = \frac{1}{2}$ $\Rightarrow \quad M^3 = \frac{1}{2}$ $\Rightarrow \quad M \approx 0.7937$	$E\left[g(X)\right] = \sum_{\forall x} g(x)P(X = x)$	$X^2 = \sum \frac{(O - i - E_i)^2}{E_i}$ where O_i are the observed frequencies and E_i are the expected frequencies. for a 2×2 contingency table we use Yate's correction $X^2_{\text{Yate's}} = \sum \frac{(\lvert O_i - E_i \rvert - 0.5)^2}{E_i}$	**Speed**: $v = r\omega$, constant along the tangent. **Acceleration**: $a = r\omega^2 = \frac{v^2}{r}$ towards the centre. **Centripetal Force:** $F = mr\omega^2 = m\frac{v^2}{r}$
Exponential distribution $f(t) = 3\mathrm{e}^{-3t}, t \geq 0$ $F(t) = 1 - \mathrm{e}^{-3t}, t \geq 0$ Mean: $\frac{1}{\mu} = \frac{1}{3}$ Variance: $\frac{1}{\mu^2} = \frac{1}{9}$	An interval generated from a sample. It is expected, before generation that the population mean μ will fall into this interval with probability p %. For a sample of size n, $\bar{x} - z \times \frac{s}{\sqrt{n}} < \mu < \bar{x} + z \times \frac{s}{\sqrt{n}}$ where s^2 is the sample variance and $z = \Phi^{-1}\left(\frac{1+p}{2}\right)$.	$I = \int_0^2 F \ \mathrm{d}t$ $= \int_0^2 2t + 3\mathrm{e}^{-2t} \ \mathrm{d}t$ $= \frac{11}{2} - \frac{3}{2\mathrm{e}^4}$	Suppose a sample of size n is taken from a distribution. We use a $t-$test if the population variance is unknown and we only know the sample variance s^2. In this case the test statistic is $T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$ and it follows a $t-$distribution with $n - 1$ degrees of freedom.	A shape on an inclined will topple if the line of action of the centre of mass lies outside of the bottom edge or corner of the shape. Suppose the plane is inclined at an angle θ to the horizontal and the coefficient of friction is μ, then the shape will slide before it topples if $\mu < (\tan(\theta))_{\text{topple}}$.
A Type I error is when a null hypothesis which is true is rejected (sometimes called a false positive). A type II error is when a null hypothesis which is false is not rejected (sometimes called a false negative)	Resolving vertically gives $T\cos(30) = 2g$ and so $T \approx 22.6$ N. Applying $F = ma$ towards the centre of the circle gives $T\sin(30) = 2 \times r \times (2\pi)^2$ and so $r \approx 0.14$ m.	Using $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$,we have $1 \times 5 + 2 \times 0 = 1 \times 2 + 2 \times v_2$ Hence, $2v_2 = 3$ and so $v_2 = 1.5$ ms^{-1}	Let R be the non gravitational resistance to motion and T be the tractive force of the car. Using $P = Fv, T = 7000$ N. Applying $F = ma$ down the plane we have that $T - R + 1200g\sin(\theta) = 1200 \times 1.2$ and so $R = 5952$.	$\mathrm{E}[X] = \int_{-\infty}^{\infty} xf(x) \ \mathrm{d}x$ $= \int_0^{\infty} x\lambda\mathrm{e}^{-\lambda x}$ $= \left[-x\mathrm{e}^{-\lambda x}\right]_0^{\infty} - \left[\frac{1}{\lambda}\mathrm{e}^{-\lambda x}\right]_0^{\infty}$ $= \frac{1}{\lambda}$ using integration by parts..
Let $X \sim \mathrm{Po}(20)$. Then $P(X = x) = \mathrm{e}^{-20}\frac{20^x}{x!}$. $P(X = 23) = 0.0669$ $\mathrm{P}(X > 25) = 1 - \mathrm{P}(X \leq 25)$ $= 0.1122$	$X + Y \sim \mathrm{N}(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$	$f(x) = \frac{1}{b-a}$ for $a < x < b$. $\mathrm{E}[X] = \frac{a+b}{2}$ $\mathrm{Var}[X] = \frac{(b-a)^2}{}$	$\mathrm{W}_{\text{A to B}} = \frac{\lambda}{2l}x_2^2 - \frac{\lambda}{2l}x_1^2$ $= \frac{50}{2 \times 6}(x_2^2 - x_1^2)$ $= \frac{25}{6}(x_2^2 - x_1^2)$	$e = \frac{\text{Speed of separation}}{\text{Speed of approach}}$ $= \frac{v_2 - v_1}{u_1 - u_2}$ where the velocities before impact are u_1 and u_2 and the velocities after the collision v_1 and v_2