AQA A-Level Further Mathematics Paper 3 Mechanics & Discrete 2022 Warmup

How do you calculate work done for a constant force F? How about for a variable force F

Describe the complete bipartite graph $K_{m,n}$. Draw $K_{3,2}$.

What are the properties of a group $(G, *)$

How do you decide if a shape on an inclined plane will topple or slide?

A body moving on a horizontal circular path of radius r with a constant angular velocity has:

speed -
acceleration -
centripetal force -

A car of mass 1200 kg is moving down a hill inclined at an angle θ where $\sin(\theta) = \frac{1}{30}$. The car is accelerating at 1.2ms^{-1} and the engine is working at a constant rate of 35 kW. Find the magnitude of the non-gravitational resistance to motion at the instant when the car is moving travelling at 5 ms^{-1}.

In the topic of collisions how do you define the coefficient of restitution?

A particle of mass 1.2 kg is acted on by a time dependent force of $F = 2t + 3e^{-2t}$. Find the impulse exerted by this force if the force is applied for 2 seconds.

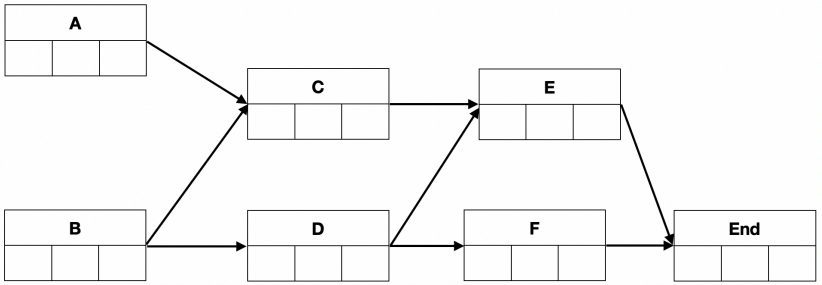
For the tasks shown in the table to the right complete the activity network in the boxes below and identify the critical activities.

Task	Immediate Predecessors	Duration (days)
A	-	3
B	-	2
C	A,B	4
D	B	2
E	C,D	5
F	C	4

Define the following terms from Game Theory:
1. zero sum game
2. play safe strategy

Four point masses are arranged in the cartesian plane. A has mass 2 kg at $(1,1)$, B has mass 3 kg at $(2,4)$, C has mass 1 kg at $(3,2)$ and D has mass 4 kg at $(4,3)$.
Find the centre of mass of this system of particles?

A particle P of mass 1kg is moving at a speed of 5ms^{-1} collides with a particle Q of mass 2kg which is at rest. Given that after the collision P moves with speed 2ms^{-1}, find the speed of Q after the collision.



An elastic string has natural length of 6m. If it is stretched by an extension x_1 it reaches point A, if it is stretched by an extension x_2 it reaches point B. If the modulus of elasticity is 50N find the work done in stretching the string from A to B.

What is a cyclic group?

State Euler's formula for connected planar graphs.

Create the Cayley table for $\times_4$ on the set $\{0,1,2,3\}$

An 3 kg object, which is initially at rest, falls 10 m freely under gravity to hit the ground. Find its final speed.

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Constant force: Work done - Force times perpendicular distance ($W = F \times d$)
Variable Force:
$$W = \int_0^d F \, \mathrm{d}x$$

The complete bipartite graph $K_{m,n}$ has each of the m vertices on one side connected to each of the n vertices on the other.

- G is a non-empty set.
- $*$ is a closed binary operation.
- $*$ is associative
- There is an identity element, e.
- Each $a \in G$ has as inverse $a^{-1} \in G$.

A shape on an inclined will topple if the line of action of the centre of mass lies outside of the bottom edge or corner of the shape.
Suppose the plane is inclined at an angle θ to the horizontal and the coefficient of friction is μ, then the shape will slide before it topples if $\mu < (\tan(\theta))_{\text{topple}}$.

Speed: $v = r\omega$, constant along the tangent.
Acceleration: $a = r\omega^2 = \frac{v^2}{r}$ towards the centre.
Centripetal Force:
$$F = mr\omega^2 = m\frac{v^2}{r}$$

Let R be the non gravitational resistance to motion and T be the tractive force of the car. Using $P = Fv$, $T = 7000$ N. Applying $F = ma$ down the plane we have that
$T - R + 1200g\sin(\theta) = 1200 \times 1.2$
and so $R = 5952$.

$$e = \frac{\text{Speed of separation}}{\text{Speed of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$
where the velocities before impact are u_1 and u_2 and the velocities after the collision v_1 and v_2

$$I = \int_0^2 F \, \mathrm{d}t = \int_0^2 2t + 3\mathrm{e}^{-2t} \, \mathrm{d}t = \frac{11}{2} - \frac{3}{2\mathrm{e}^4}$$

Critical activities: E, C, A

Task	Immediate Predecessors	Duration (days)
A	-	3
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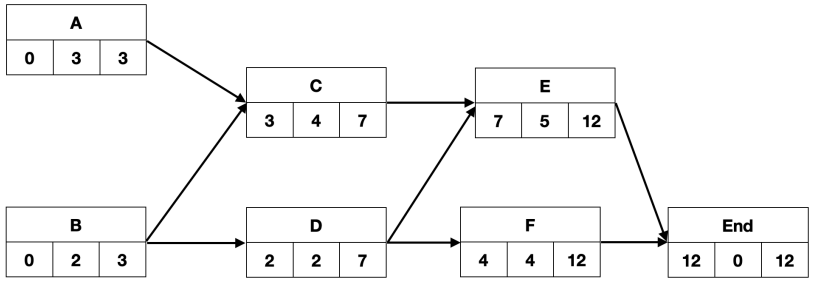
1. In a zero-sum game, the sum of the gains made by the players on each play is zero.
2. A play-safe strategy gives the best guaranteed outcome regardless of what the other player does.

(2.7,2.8)

Using $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$,we have
$1 \times 5 + 2 \times 0 = 1 \times 2 + 2 \times v_2$
Hence, $2v_2 = 3$ and so
$v_2 = 1.5$ ms^{-1}



$$W_{A \text{ to } B} = \frac{\lambda}{2l}x_2^2 - \frac{\lambda}{2l}x_1^2 = \frac{50}{2 \times 6}(x_2^2 - x_1^2) = \frac{25}{6}(x_2^2 - x_1^2)$$

A cyclic group is formed if, for example, only rotational symmetries are considered. Cyclic groups are groups that can be generated by a single element.

Euler's formula for connected planar graphs states that $F + V = E + 2$ where F is the number of faces, V is the number of vertices and E is the number of edges.

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

$$mgh = \frac{1}{2}mv^2$$
$$\Rightarrow \quad 3 \times 9.8 \times 10 = \frac{1}{2} \times 3 \times v^2$$
$$\Rightarrow quadv^2 = 196$$
$$\Rightarrow \quad v = 14$$
so $v^2 = 14$ ms^{-1}