AQA A-Level Further Mathematics Paper 3 Discrete & Statistics 2022 Warmup

Describe the complete bipartite graph $K_{m,n}$.
Draw $K_{3,2}$.

A CRV X, has probability density function given by
$f(x) = \begin{cases} 3x^a; & 0 \le x \le 1 \\ 0; & \text{otherwise} \end{cases}$
Find the constant a and the median value M, of X.

The expected value of a function $g(X)$ of a discrete random variable X is given by:

What is the test statistic for a Chi Squared test?
What modification is needed for a 2×2 contingency table?

A cyclic group is formed if, for example, only rotational symmetries are considered. Cyclic groups are groups that can be generated by a single element.

Random events occur at a rate of 3 per minute.
a) Write the probability density function $f(t)$ and the cumulative density function $F(t)$ for the random variable T, the waiting time in minute between events.
b) What is the mean and variance of T.

What is a $p\,\%$ confidence interval?

When would you use a $t-$test? And what is the formula for the test statistic?

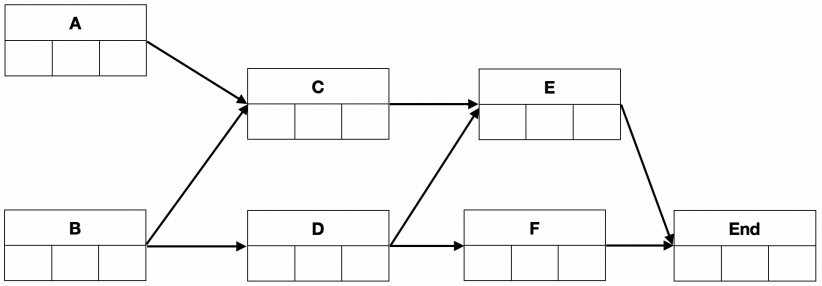
For the tasks shown in the table to the right complete the activity network in the boxes below and identify the critical activities.

Task	Immediate Predecessors	Duration (days)
A	-	3
B	-	2
C	A,B	4
D	B	2
E	C,D	5
F	C	4

What are Type I and Type II errors?

Prove that the exponential distribution $f(x) = \lambda \mathrm{e}^{-\lambda x}$, with $x \ge 0$ has a mean of $\frac{1}{\lambda}$.

Define the following terms from Game Theory:
1. zero sum game
2. play safe strategy



A Geiger counter detects radioactive decays at a mean rate of 20 per minute. Find the probability that in a given, randomly chosen minute, there are
i) 23 decays
ii) More than 25 decays

If X and Y are Poisson random variables then what is the distribution of $X + Y$?

For the rectangular distribution $X \sim \mathrm{Rect}(a, b)$ what is the probability density function, the mean and the variance?

Create the Cayley table for $\times_4$ on the set $\{0,1,2,3\}$

State Euler's formula for connected planar graphs.

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The complete bipartite graph $K_{m,n}$ has each of the m vertices on one side connected to each of the n vertices on the other.

$$\int_0^1 3x^a \, \mathrm{d}x = 1 \quad \Rightarrow \quad a = 2$$

$$\int_0^M 3x^2 \, \mathrm{d}x = \frac{1}{2}$$

$$\Rightarrow \quad M^3 = \frac{1}{2}$$

$$\Rightarrow \quad M \approx 0.7937$$

$$E\left[g(X)\right] = \sum_{\forall x} g(x)P(X = x)$$

$X^2 = \sum \frac{(O - i - E_i)^2}{E_i}$ where O_i are the observed frequencies and E_i are the expected frequencies.
for a 2×2 contingency table we use Yate's correction

$$X^2_{\text{Yate's}} = \sum \frac{(|O_i - E_i| - 0.5)^2}{E_i}$$

What is a cyclic group?

Exponential distribution
$f(t) = 3\mathrm{e}^{-3t}, t \geq 0$
$F(t) = 1 - \mathrm{e}^{-3t}, t \geq 0$
Mean: $\frac{1}{\mu} = \frac{1}{3}$
Variance: $\frac{1}{\mu^2} = \frac{1}{9}$

An interval generated from a sample. It is expected, before generation that the population mean μ will fall into this interval with probability $p\,\%$.
For a sample of size n,
$\bar{x} - z \times \frac{s}{\sqrt{n}} < \mu < \bar{x} + z \times \frac{s}{\sqrt{n}}$
where s^2 is the sample variance and
$z = \Phi^{-1}\left(\frac{1+p}{2}\right)$.

Suppose a sample of size n is taken from a distribution. We use a $t-$test if the population variance is unknown and we only know the sample variance s^2. In this case the test statistic is $T = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$ and it follows a $t-$distribution with $n - 1$ degrees of freedom.

Critical activities: E, C, A

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A	-	3
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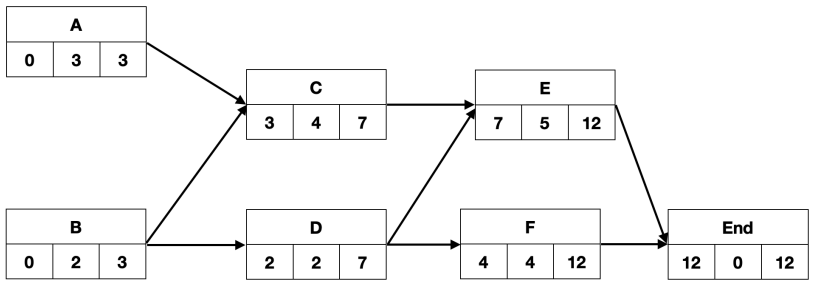
A Type I error is when a null hypothesis which is true is rejected (sometimes called a false positive). A type II error is when a null hypothesis which is false is not rejected (sometimes called a false negative)

$$\begin{aligned}\mathrm{E}[X] &= \int_{-\infty}^{\infty} x f(x) \, \mathrm{d}x \\ &= \int_0^{\infty} x\lambda \mathrm{e}^{-\lambda x} \\ &= \left[-x\mathrm{e}^{-\lambda x}\right]_0^{\infty} - \left[\frac{1}{\lambda}\mathrm{e}^{-\lambda x}\right]_0^{\infty} \\ &= \frac{1}{\lambda}\end{aligned}$$

using integration by parts..

1. In a zero-sum game, the sum of the gains made by the players on each play is zero.



Let $X \sim \mathrm{Po}(20)$. Then
$P(X = x) = \mathrm{e}^{-20}\frac{20^x}{x!}$.
$P(X = 23) = 0.0669$
$\mathrm{P}(X > 25) = 1 - \mathrm{P}(X \leq 25)$
$= 0.1122$

$$X + Y \sim \mathrm{Po}(\lambda_1 + \lambda_2)$$

1. A play-safe strategy gives the best guaranteed outcome regardless of what the other player does.

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Euler's formula for connected planar graphs states that $F + V = E + 2$ where F is the number of faces, V is the number of vertices and E is the number of edges.