AQA A-Level Further Mathematics Warmup - Paper 22022

| Let $z=3+4 \mathrm{i}$ and $w=2+5 \mathrm{i}$. Find $z w$. | Use the Maclaurin series of $\cos (x)$ to find a series expansion for $\cos \left(x^{2}+3 x\right)$ up to the term in $x^{3}$. | Sketch $\frac{x^{2}}{49}-\frac{y^{2}}{25}=1$ | Find the mean value of $f(x)=\cosh (x)$ over the interval $[\ln (2), \ln (4)]$. | Find the eigenvalues and eigenvectors of the matrix $\left(\begin{array}{ll} 1 & 3 \\ 3 & 1 \end{array}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| The quadratic equation $3 x^{2}+10 x+5=0$ has roots $\alpha, \beta$ find the quadratic equation with roots $\frac{\alpha+1}{2}, \frac{\beta+1}{2}$. | What is the integrating factor when solving $\frac{\mathrm{d} y}{\mathrm{~d} x}+y=\mathrm{e}^{2 x} ?$ | How do you find the volume generated when the function $f(x)$, between $x=a$ and $x=b$ is rotated $2 \pi$ radians around the $x$ axis? | Find the equations of the asymptotes and vertices of the hyperbola $\frac{(x-3)^{2}}{25}-\frac{(y+1)^{2}}{16}=1$ | Give a suitable concluding statement for a proof by induction. |
| Sketch $y=\frac{3 x-2}{x+4}$ | Use the Mid-ordinate rule to approximate $\int_{2}^{4} \ln (x) \mathrm{d} x \text { with } 4 \text { strips. }$ | Prove by induction that $11^{n}-6$ is divisible by 5 for all positive integer $n$. | Let $z=3+4 \mathrm{i}$ and $w=2+5$ i. Find $\frac{z}{w^{*}}$. | Solve $x \frac{d y}{d x}+2 y=10 x^{2}$ |
| Find the characteristic polynomial of the matrix $A=\left(\begin{array}{ll} 1 & 3 \\ 4 & 1 \end{array}\right)$ | Sketch $y=\sinh (x)$ | State Viète's formulae for the cubic equation $a x^{3}+b x^{2}+c x+d=0$ with roots $\alpha, \beta$ and $\gamma$. | What is the matrix representing a rotation by $60^{\circ}$ anticlockwise followed by a reflection in the line $y=-x$ ? | Find the volume of revolution when $y=x^{3}-2 x^{2}$ is rotated about the $x$-axis between $x=1$ and $x=3$. |

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| $-14+23 \mathrm{i}$ | $\cos \left(x^{2}+3 x\right)=1-\frac{9}{2} x^{2}-3 x^{3}+O\left(x^{4}\right)$ |  | $\frac{9}{8(\ln (4)-\ln (2))}$ | $\begin{aligned} & \lambda_{1}=4 \text { with } \mathbf{v}_{1}=\binom{1}{1} \\ & \lambda_{2}=-2 \text { with } \mathbf{v}_{2}=\binom{-1}{1} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: |
| Use the substitution $x=2 w-1$ to obtain $12 w^{2}+8 w-2$. | $\mathrm{IF}=\mathrm{e}^{\int 1 \mathrm{~d} x}=\mathrm{e}^{x}$ | $V=\pi \int_{a}^{b} y^{2} \mathrm{~d} x$ | Asymptotes are $y=\frac{4}{5} x-\frac{17}{5}$ and $y=-\frac{4}{5} x+\frac{7}{5}$. Vertices have coordinates $(-2,-1)$ and $(8,-1)$ | As the statement is true for $n=1$ and we have shown that if it holds for $n=k$ then it also holds for $n=k+1$ we conclude that the statement must be true for all $n \geq 1$ by the principle of mathematical induction. |
|  | $\approx 2.1548$ | Proof | $\frac{1}{29}(-14+23 i)$ | $I F=x^{2}$ <br> General solution is $y=\frac{5}{2} x^{2}+\frac{C}{x^{2}}$ |
| $p(A)=\lambda^{2}-2 \lambda-11$ |  | $\begin{aligned} & \alpha+\beta+\gamma=\frac{-b}{a} \\ & \alpha \beta+\alpha \gamma+\beta \gamma=\frac{c}{a} \\ & \alpha \beta \gamma=-\frac{d}{a} \end{aligned}$ | $\begin{array}{r} \left(\begin{array}{cc} 0 & -1 \\ -1 & 0 \end{array}\right)\left(\begin{array}{cc} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{array}\right) \\ =\left(\begin{array}{cc} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{array}\right) \end{array}$ | $\frac{2158 \pi}{105}$ |

